

Dicke Model for Quantum Hall Systems

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Quantum Hall (QH) systems consist of many-body electron and nuclear spins. They are coupled so weakly through the hyperfine interaction that normally electron spin dynamics are scarcely affected by the nuclear spins. The dynamics of the QH systems, however, may drastically change when the nuclear spins interact with low-energy collective excitation modes of the electron spins. We theoretically investigate the nuclear-electron spin interaction in the QH systems as hybrid quantum systems driven by the hyperfine interaction. In particular, we study the interaction between the nuclear spins and the Nambu-Goldstone (NG) mode with the linear dispersion relation associated with the U(1) spin rotational symmetry breaking. We show that such an interaction is described as nuclear spins collectively coupled to the NG mode, and can be effectively described by the Dicke model. Based on the model we suggest that various collective spin phenomena realized in quantum optical systems also emerge in the QH systems.

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I. INTRODUCTION

Quantum Hall (QH) systems exhibit fascinating macroscopic quantum phenomena [1, 2]. For instance, various low-energy electron coherent phenomena in terms of spin and/or pseudospin (layer) internal degrees of freedom are realized. Research interests for the QH physics are not limited to the electron spin physics. However, the electron-nuclear spin dynamics has not yet attracted much attention. For instance, the GaAs semiconductor with the *s*-type conduction band has a large natural abundance of nuclear spins with the 3/2 nuclear spin angular momentum (⁶⁹Ga, ⁷¹Ga, and ⁷⁵As). Although the nuclear spins interact with the electron spins mainly through the Fermi contact hyperfine interaction, it is usually so weak that electron transport properties are not affected by this interaction. Thus, previous studies of QH physics have mainly been focused on the electron spin physics, whereas the nuclear spins are utilized merely as a tool to investigate the electron magnetic properties [3].

The above situation may change when the nuclear spins interact with low-energy excitation modes of electron spins such as Nambu-Goldstone (NG) modes. These modes appear in the canted antiferromagnetic (CAF) phase [4–12], which is the most interesting one among the three phases of the total filling factor $\nu = 2$ bilayer QH systems; the ferromagnetic phase, the spin-singlet phase and the CAF phase. In the CAF phase, the antiferromagnetic correlations between the electron spins in one of the layers (abridged as front layer) and those in the other layer (abridged as back layer) are generated, and the associated linear dispersing NG mode emerges. A new physics is expected to emerge in this phase. In the related context, the nuclear spin relaxation was experimentally estimated by using the resistivity-detected nuclear spin relaxation measurement [10], where the longitudinal resistance R_{xx} is used as a measure of the nuclear-spin polarization. It has been shown that the nuclear-spin relaxation time in the CAF phase is the shortest compared with those in the other two phases (the ferromagnetic phase and the spin-singlet phase). More recent experiment [12], where the spatial nuclear-spin polarization distribution was recorded after the exposure to the CAF phase, showed a sudden change in the nuclear-spin polarization distribution from the initial one. These experimental results suggest a unique many-body interaction between electron and nuclear spin systems.

It is worthwhile to investigate the nuclear spin physics mediated by the hyperfine interaction in the QH systems also from the following three perspectives: First, nonequilibrium phenomena of nuclear spins in the QH systems are still less understood to date. Second, we expect a rich variety of many body effects as well as cooperative phenomena in terms of a nuclear-spin ensemble driven by the magnetic properties of the QH systems, just like the superradiance phenomena in optical cavity quantum electrodynamics systems composed of atomic Bose-Einstein condensate [13] and in quantum dots [14, 15]. Third, the electron-nuclear spin hybrid system is a candidate for a spin-base quantum information processing and computing with a coherent manipulation [3, 16].

In this paper, for the first step of studying the electron-nuclear spin dynamics in the QH systems as hybrid quantum systems (the schematic illustration is presented in FIG. 1), we theoretically study the interaction between the nuclear spins and the linear dispersing NG mode associated with the U(1) spin rotational symmetry breaking. To make a detailed analysis and clearly understand its physics behind, we focus on the CAF phase in the $\nu = 2$ bilayer QH system. We show that the NG mode couples with collective nuclear spins through the hyperfine interaction in the long-wavelength limit. As a result, such an interaction is effectively described by a Dicke model [17–20] with a continuous-mode, which has been extensively studied in the quantum

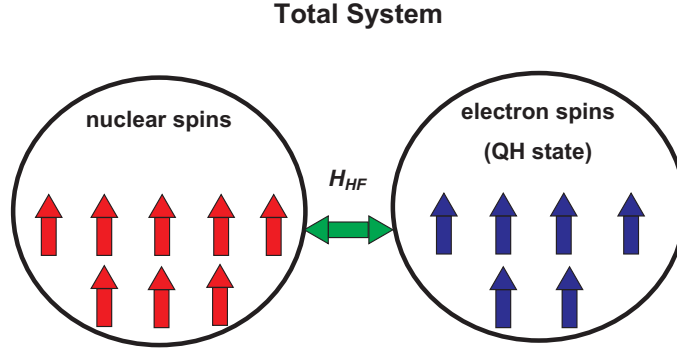


FIG. 1: A schematic illustration of “hybrid” quantum Hall (QH) systems. They consist of electron spins in the QH state and a large ensemble of nuclear spins in host crystals. They are coupled through the hyperfine interaction H_{HF} , and therefore, the total system can be regarded as a hybrid QH systems.

optics. It is interesting that the interaction between nuclear spins and the linear dispersing NG mode mediated by the hyperfine interaction in the QH systems can be described by the same model as the two-level atomic system coupled with photonic modes.

Our analysis is not only valid for the nuclear spin-NG mode interaction in the CAF phase, but also for other QH systems where a linear dispersing NG mode is present due to the U(1) spin rotational symmetry breaking. For example, the skyrmion crystal formation in the monolayer QH system in the vicinity of the total filling factor $\nu = 1$ [21] may belong to this category, where a linear dispersing spin wave emerges and is expected to enhance the nuclear spin relaxation rate.

This paper is organized as follows. In Sec. II, we analyze the hyperfine interaction in the QH systems. Based on this analysis, in Sec. III we derive the Dicke model as the effective Hamiltonian describing the interaction between the nuclear spins and the linear dispersing NG mode through the hyperfine interaction. It is the main result of this paper.

II. HYPERFINE INTERACTION IN THE QH SYSTEMS

We first analyze the hyperfine interaction in QH systems to derive the interaction Hamiltonian between the nuclear spins and the NG mode. As shown in FIG. 1, the QH systems consist of many-body nuclei and many-body electrons in the two-dimensional xy plane. We may assume that all electrons are in the s -type conduction band. A high magnetic field is applied perpendicular to the plane, $\mathbf{B} = (0, 0, -B_{\perp})$ with $B_{\perp} > 0$.

The interaction between nuclear and electron spins in the QH state is described by the contact hyperfine interaction [22–24],

$$\begin{aligned} H_{\text{HF}} &= \frac{2\mu_0\gamma_e\gamma_n\hbar^2}{3} \sum_{i=1}^N \mathbf{I}_i \cdot \sum_{j=1}^{N_e} |u(\mathbf{x}_j, z_j)|^2 \mathbf{S}_j \delta(\mathbf{X}_i - \mathbf{x}_j, Z_i - z_j) \\ &= \frac{2\mu_0\gamma_e\gamma_n\hbar^2}{3} \sum_{i=1}^N |u(\mathbf{X}_i, Z_i)|^2 \mathbf{I}_i \cdot \mathbf{S}(\mathbf{X}_i, Z_i), \end{aligned} \quad (1)$$

where \mathbf{S}_j is the electron spin at (\mathbf{x}_j, z_j) , \mathbf{I}_i the nuclear spin at (\mathbf{X}_i, Z_i) assuming spin 1/2, and $\mathbf{S}(\mathbf{X}_i, Z_i)$ the three dimensional electron spin density. At sufficiently low temperature electrons are confined within the lowest energy level, and hence the motion of electrons along the z direction is frozen. We may approximate the quantum well by the system where electrons are located at the center of the well, that is, $z_j = z_0$ for all j , and interact with nuclear spins at $Z_i = z_0$ for all i . Then we may set $\mathbf{S}(\mathbf{X}_i, Z_i) \simeq \mathbf{S}(\mathbf{X}_i) L_z^{-1}$ for $-L_z/2 < Z_i < L_z/2$ with L_z the width of quantum well and $\mathbf{S}(\mathbf{X}_i)$ the two dimensional electron spin density. The quantities γ_e (γ_n), μ_0 , $u(\mathbf{X}_i, z_0)$, N , and N_e are the gyromagnetic ratio for electron (nucleon), the permeability of vacuum magnetic constant, the Bloch amplitude at (\mathbf{X}_i, z_0) , the total number of polarized nuclear spins, and the total electron number, respectively. Since the Bloch amplitude is a periodic function with respect to the nuclear spin separation, we can set $|u(\mathbf{X}_i, z_0)|^2 = \eta = \text{const}$. Then the hyperfine interaction (1) is rewritten as

$$H_{\text{HF}} = A \sum_{i=1}^N \mathbf{I}_i \cdot \mathbf{S}(\mathbf{X}_i), \quad \text{with} \quad A = \frac{2\mu_0\gamma_e\gamma_n\hbar^2\eta}{3L_z}. \quad (2)$$

The values of η for Ga and As are given by $\eta_{\text{Ga}} = 2.7 \times 10^3$ and $\eta_{\text{As}} = 4.5 \times 10^3$, respectively [25]. Here we take $\eta = 10^3$.

The hyperfine coupling is weak compared with the Landau-level energy as well as the thermal energy. It is reasonable to assume that the electronic ground state is unchanged by the hyperfine interaction, and to replace the electron spin density in Eq. (2) by the classical spin density $\mathbf{S}^{\text{cl}}(\mathbf{X}_i) = \langle \phi_{\text{QH}} | \mathbf{S}(\mathbf{X}_i) | \phi_{\text{QH}} \rangle$, with $|\phi_{\text{QH}}\rangle$ denoting the QH state. We express the hyperfine interaction in terms of normalized spin density defined by $\mathbf{S}^{\text{cl}}(\mathbf{X}_i) = \rho_{\Phi}^{-1} \mathbf{S}^{\text{cl}}(\mathbf{X}_i)$, where $\rho_{\Phi} = \rho_0/\nu = 1/2\pi l_B^2$ is the density of states of the Landau sites with ρ_0 the total electron density, ν the total Landau filling factor, and l_B the magnetic length. Eq. (2) becomes

$$H_{\text{HF}} = \tilde{g} \sum_{i=1}^N \mathbf{I}_i \cdot \mathbf{S}^{\text{cl}}(\mathbf{X}_i), \quad (3)$$

where $\tilde{g} = A\rho_{\Phi}$. The Hamiltonian (3) describes the hyperfine interaction between the nuclear spins and the electron spins in the QH state. The order of the coupling \tilde{g} in (3) is estimated by setting $\mu_0 = 4\pi \times 10^{-7} \text{ N}\cdot\text{A}^{-2}$, $\gamma_e = 1.761 \times 10^{11} \text{ rad/s}\cdot\text{T}$, $\gamma_n = 10 \times 10^7 \text{ rad/s}\cdot\text{T}$, $\rho_0 = 1 \times 10^{15} \text{ m}^{-2}$, $L_z = 10^{-8} \text{ m}$, $\eta = 10^3$, and $\hbar = 1.0546 \times 10^{-34} \text{ J}\cdot\text{s}$, and we have $\tilde{g}/\hbar \sim 100 \text{ rad}\cdot\text{kHz/T}$. It is much smaller compared with the Larmor frequency $\omega_s \sim 10 \text{ rad}\cdot\text{MHz/T}$.

III. DICKE MODEL IN THE QH SYSTEMS

We next derive the interaction Hamiltonian between the nuclear spins and the NG mode. We show that collective nuclear spins couple with the NG mode in the long-wavelength limit, and furthermore that it is effectively described by the Dicke model.

A. Electron Spin Configuration and Effective Hamiltonian for the NG mode

When the U(1) spin rotational symmetry is spontaneously broken around the z -axis, the in-plane component of the classical electron spin density is expressed as

$$\mathbf{S}^{\text{cl},x}(\mathbf{x}) = S \cos(\vartheta_0 + \delta\vartheta(\mathbf{x})), \quad \mathbf{S}^{\text{cl},y}(\mathbf{x}) = S \sin(\vartheta_0 + \delta\vartheta(\mathbf{x})), \quad (4)$$

where S is a constant in the range $0 < |S| < 1$. In the ground state the electron spins are in a spatially homogeneous configuration with a fixed orientation angle ϑ_0 of the in-plane spin component. The fluctuation field $\delta\vartheta(\mathbf{x})$ is the associated NG mode. For the z -component, it is assumed that the fluctuation of the electron spin density is negligible, because it is gapped.

We expand the above spin densities in terms of $\delta\vartheta(\mathbf{x})$ up to the first order and substitute it to the hyperfine interaction (3). The zeroth order terms with respect to $\delta\vartheta(\mathbf{x})$ are

$$H_{\text{HF}}^{(0)} = \left(\tilde{g} S \sum_i (I_i^x \cos \vartheta_0 + I_i^y \sin \vartheta_0) \right) + \tilde{g} \sum_i I_i^z S_z^g, \quad (5)$$

where S_z^g is the ground-state expectation value of the z component of the normalized electron spin density satisfying $0 < |S_z^g| < 1$. The first term describes the in-plane magnetic field induced by the hyperfine interaction, which nuclear spins experience, while the second term generates the Knight shift $K_s\omega_s$. These two terms have the same order of magnitude as the coupling $\tilde{g}/\hbar \sim 100 \text{ rad}\cdot\text{kHz/T}$, which was mentioned in the previous section, i.e., $K_s\omega_s \sim 100 \text{ rad}\cdot\text{kHz/T}$. Here we note that this value is comparable with the experimental result reported in [10], i.e., $(K_s\omega_s)^{\text{Exp}} \sim 10 \text{ rad}\cdot\text{kHz}$. We can drop these terms because they are negligible compared with the nuclear-spin Larmor frequency, which is $\omega_s \sim 10 \text{ rad}\cdot\text{MHz/T}$.

The first-order terms in $\delta\vartheta(\mathbf{x})$ describe the interaction between the nuclear spins and the NG mode,

$$H_{\text{HF}}^{(1)} = g \sum_i (-I_i^x \sin \vartheta_0 + I_i^y \cos \vartheta_0) \delta\vartheta(\mathbf{x}), \quad \text{with} \quad g = \tilde{g} S. \quad (6)$$

To understand the situation more clearly, we present an example, the CAF phase in the $\nu = 2$ bilayer QH state. As presented in FIG. 2, electron spins have ferromagnetic correlations in each layer, whereas they have antiferromagnetic correlations between the two layers. The electron spin configuration in the front (back) layer $S_a^{f(b)}$ ($a = x, y, z$) is described as

$$\mathbf{S}_x^f = -\mathbf{S}_x^b = S \cos \vartheta_0, \quad \mathbf{S}_y^f = -\mathbf{S}_y^b = S \sin \vartheta_0, \quad \mathbf{S}_z^f = \mathbf{S}_z^b = S_z^{\text{caf}}, \quad (7)$$

where $S_a^{f(b)}$ ($a = x, y$) and $S_z^{f(b)}$ are the in-plane and z components of the electron spin in the front (back) layer, respectively. Here S and S_z^{caf} satisfy the conditions $0 < |S| < 1$ and $0 < |S_z^{\text{caf}}| < 1$. As a result, spins are canted coherently. By focusing on the in-plane component, electron spins orient homogeneously characterized by an angle ϑ_0 . Although the ground state energy

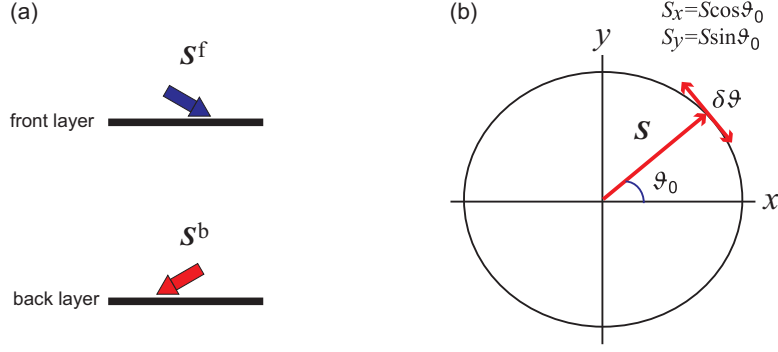


FIG. 2: Electron-spin configuration in the CAF phase. (a) Electron spins have ferromagnetic correlation in each layer, while antiferromagnetic correlation between the two layers. Consequently, electron spins are aligned (canted) coherently. (b) The electron spin for in-plane component in the front layer. We denote this plane as xy plane whereas the z axis indicate the direction perpendicular to the xy plane. The electron spins are in a spatially homogeneous configuration, characterized by the orientation angle ϑ_0 . The fluctuation mode $\delta\vartheta$ is the NG mode.

does not depend on ϑ_0 , the ground state itself does, reflecting that the CAF state is the U(1) spin rotational symmetry broken state around z axis. The small fluctuation mode $\delta\vartheta$ is the corresponding NG mode. We present the explicit formula of (7) for the case of the CAF phase in Appendix A.

The intriguing feature of the NG mode is that it has a linear dispersion in the CAF phase. We denote the Fourier transform of $\delta\vartheta(\mathbf{x})$ as $\delta\vartheta_{\mathbf{k}}$,

$$\delta\vartheta_{\mathbf{k}} = \int \frac{d^2x}{2\pi} e^{-i\mathbf{k}\cdot\mathbf{x}} \delta\vartheta(\mathbf{x}), \quad (8)$$

and introduce the canonical conjugate variable $\delta\sigma_{\mathbf{k}}$ satisfying $[\delta\sigma_{\mathbf{k}}, \delta\vartheta_{\mathbf{k}'}^\dagger] = i\delta(\mathbf{k} - \mathbf{k}')$. By making the momentum expansion, the effective Hamiltonian for the NG mode has the following form,

$$H_R = \int d^2k \left[(a + b\mathbf{k}^2) \delta\sigma_{\mathbf{k}}^\dagger \delta\sigma_{\mathbf{k}} + (c\mathbf{k}^2) \delta\vartheta_{\mathbf{k}}^\dagger \delta\vartheta_{\mathbf{k}} \right], \quad (9)$$

where a , b and c are positive constants. This Hamiltonian is diagonalized by introducing another set of canonical variables,

$$r_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left(\sqrt{G_{\mathbf{k}}} \delta\sigma_{\mathbf{k}} + i \frac{1}{\sqrt{G_{\mathbf{k}}}} \vartheta_{\mathbf{k}} \right), \quad r_{\mathbf{k}}^\dagger = \frac{1}{\sqrt{2}} \left(\sqrt{G_{\mathbf{k}}} \delta\sigma_{\mathbf{k}}^\dagger - i \frac{1}{\sqrt{G_{\mathbf{k}}}} \vartheta_{\mathbf{k}}^\dagger \right), \quad (10)$$

satisfying $[r_{\mathbf{k}}, r_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$, where

$$G_{\mathbf{k}} = \left(\frac{a + b\mathbf{k}^2}{c\mathbf{k}^2} \right)^{\frac{1}{2}}. \quad (11)$$

Now the effective Hamiltonian (9) is diagonalized as

$$H_R = \int d^2k E_{\mathbf{k}} r_{\mathbf{k}}^\dagger r_{\mathbf{k}}, \quad \text{with} \quad E_{\mathbf{k}} = 2\sqrt{c\mathbf{k}^2(a + b\mathbf{k}^2)} \approx 2\sqrt{ac}|\mathbf{k}|, \quad (12)$$

with $E_{\mathbf{k}}$ a linear dispersion relation for the NG mode. We present the explicit formulas of Eqs. (9), (11), and (12) for the case of the CAF phase in Appendix B: See Eqs. (B4), (B7), and (B10), respectively.

B. Dicke Model

We now describe the hyperfine interaction Hamiltonian (6) in terms of the NG mode $r_{\mathbf{k}}, r_{\mathbf{k}}^\dagger$ and the nuclear spins $I_i^{x,y}$. Using Eqs. (8) and (10), we obtain

$$H_{\text{HF}} = \frac{g}{2} \sum_{i=1}^N (\tilde{I}_i^+ + \tilde{I}_i^-) \int \frac{d^2k}{2\pi} \left(\kappa_{\mathbf{X}_i, \mathbf{k}} r_{\mathbf{k}} + \kappa_{\mathbf{X}_i, \mathbf{k}}^* r_{\mathbf{k}}^\dagger \right),$$

$$\kappa_{\mathbf{X}_i, \mathbf{k}} = \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{i(\mathbf{k} \cdot \mathbf{X}_i - \frac{\pi}{2})}, \quad \kappa_{\mathbf{X}_i, \mathbf{k}}^* = \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{-i(\mathbf{k} \cdot \mathbf{X}_i - \frac{\pi}{2})}. \quad (13)$$

where $\tilde{I}^\pm = e^{\mp i(\vartheta_0 + \pi/2)} I^\pm$ ($I^\pm = I^x \pm iI^y$) is the rotated in-plane nuclear spin. In the rest of this paper we just write \tilde{I}^\pm as I^\pm . To derive (13), we used the relations $G_{\mathbf{k}} = G_{-\mathbf{k}}$, $\delta\sigma_{\mathbf{k}}^\dagger = \delta\sigma_{-\mathbf{k}}$ and $\vartheta_{\mathbf{k}}^\dagger = \vartheta_{-\mathbf{k}}$. They follow from the fact that $G_{\mathbf{k}}$ are even function of \mathbf{k} , and $\delta\sigma(\mathbf{x}) = \int (d^2k/2\pi) e^{-i\mathbf{k} \cdot \mathbf{x}} \delta\sigma_{\mathbf{k}}^\dagger$ and $\delta\vartheta(\mathbf{x})$ are real fields. Using the rotating-wave approximation for (13), we obtain

$$H_{\text{SR}} = \frac{g}{2} \sum_{i=1}^N (I_i^+ R_i + I_i^- R_i^\dagger),$$

$$R_i = \int \frac{d^2k}{2\pi} \kappa_{\mathbf{X}_i, \mathbf{k}} r_{\mathbf{k}}, \quad R_i^\dagger = \int \frac{d^2k}{2\pi} \kappa_{\mathbf{X}_i, \mathbf{k}}^* r_{\mathbf{k}}^\dagger. \quad (14)$$

Furthermore, the i dependence disappears from R_i and R_i^\dagger in the long wave-length limit $e^{i\mathbf{k} \cdot \mathbf{X}_i} \approx 1$. We may rewrite (14) as

$$H_{\text{SR}} = \frac{g}{2} \sum_{i=1}^N (I_i^+ R + I_i^- R^\dagger) = \frac{g}{2} (J^+ R + J^- R^\dagger),$$

$$R = \int \frac{d^2k}{2\pi} \kappa_{\mathbf{k}} r_{\mathbf{k}}, \quad R^\dagger = \int \frac{d^2k}{2\pi} \kappa_{\mathbf{k}}^* r_{\mathbf{k}}^\dagger,$$

$$\kappa_{\mathbf{k}} = \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{-i\frac{\pi}{2}}, \quad \kappa_{\mathbf{X}_i, \mathbf{k}}^* = \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{i\frac{\pi}{2}}. \quad (15)$$

Indeed, as the NG mode has a long wavelength, the approximation $e^{i\mathbf{k} \cdot \mathbf{X}_i} \approx 1$ is valid in this system. For instance, in the case of the CAF phase the wavelength of the NG mode becomes $\lambda_s \sim 10^7 \text{ \AA}$ for $E_{\mathbf{k}} = \hbar\omega_s$ with $\omega_s \sim 10 \text{ rad}\cdot\text{MHz/T}$, where $E_{\mathbf{k}} = \gamma|\mathbf{k}|$ ($\gamma = 2\sqrt{ac} > 0$) is the linear dispersion relation for the NG mode. The value of λ_s is about the same as the sample size $L \sim 100 \mu\text{m}$ (see Eq. (B11)). Thus it is a good approximation at this energy scale. If the dispersion for the NG mode were quadratic with the same coefficient as the linear one, the wavelength at $E_{\mathbf{k}} = \hbar\omega_s$ would be around $\lambda_s \sim 10^3 \text{ \AA}$, which is much smaller than the sample size. Such a case, the long-wavelength limit is not a good approximation so that the interaction Hamiltonian H_{SR} cannot be expressed in terms of the NG mode and the collective spin.

As a result, the interaction between the nuclear spins and the linear dispersing NG mode is described as an interaction between the collective nuclear spin $J = \sum_i I_i$ and the NG mode R with the coupling constant g in the long-wavelength limit.

The other relevant terms are the nuclear-spin Hamiltonian H_S describing the Larmor precession,

$$H_S = -\hbar\gamma_n B_z \sum_{i=1}^N I_i^z = \hbar\omega_s \sum_{i=1}^N I_i^z = \hbar\omega_s J^z. \quad (16)$$

The total effective Hamiltonian $H = H_S + H_R + H_{\text{SR}}$ is given by

$$H = \hbar\omega_s J^z + \int d^2k E_{\mathbf{k}} r_{\mathbf{k}}^\dagger r_{\mathbf{k}} + \frac{g}{2} (J^+ R + J^- R^\dagger). \quad (17)$$

Consequently, the interaction between the nuclear spins and the linear dispersing NG mode mediated by the hyperfine interaction is described effectively by the Dicke model [17–20] with a continuous-mode, where the collective spin operator J^a ($a = x, y, z$) with its magnitude $N/2$ interacts with the NG mode with the coupling g . It is interesting that the nuclear spin-NG mode interaction mediated by the hyperfine interaction in the QH systems can be described by the analogue model as the two-level atomic systems surrounded by the electromagnetic field in the vacuum: Nuclear-spins 1/2 correspond to the two-level atoms, whereas the NG mode corresponds to the electromagnetic field in the vacuum [20]. For an explicit derivation of this Dicke model in the case of the CAF phase, see Appendix C.

IV. CONCLUSION

In this paper, we have presented the theoretical studies on the interaction between the nuclear spins and the linear dispersing NG mode due to the spin U(1) rotational symmetry breaking in the QH systems mediated by the hyperfine interaction. Since the NG mode has a long wavelength, the nuclear spins couple collectively with the NG mode. Consequently, the nuclear spin-NG mode interaction can be described effectively by the Dicke model with a continuous-mode. This physics could be captured by regarding the QH systems as hybrid quantum systems comprise of electron and nuclear spins. In this paper, we focused on the CAF phase in $\nu = 2$ bilayer QH systems and demonstrated that the interaction between the nuclear spins and the NG mode is described by the above Dicke model. This Dicke model must be also applied to QH systems with the linear dispersing NG mode due to the spin U(1) rotational symmetry breaking in general, for instance, the nuclear spin-NG mode interaction in the vicinity of $\nu = 1$ monolayer QH systems in the presence of the skyrmion crystal formation.

We would like to emphasize that the Dicke model derived in this paper can lead to new directions for the study of QH physics, which cannot be obtained solely from the perspectives of the solid state physics. Combining the perspectives of solid state physics with these from quantum optics, we may reveal rich and new phenomena in the QH systems. For instance, the realization of phenomena similar to those in the quantum optical systems, for example, superradiance in terms of nuclear spins in the QH systems might be one possibility. The study of the spin-boson dynamics inherent to the QH system might be another interesting direction.

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Appendix A: Order Parameters in the CAF phase

We present the derivation of the Dicke model by constructing the Hamiltonian H_R for the NG mode and the interaction Hamiltonian H_{SR} between the nuclear spin and the NG mode in the case of the CAF phase. To derive the Dicke model, we make a concise review of the effective Hamiltonian density for the ground state and the NG modes in bilayer QH systems [2, 8, 11]. We start with the discussion on the phase structure as well as the spin density configuration, and the associated NG modes at $\nu = 2$. Then we present the effective Hamiltonian density for the linear dispersing NG mode in the CAF phase.

In the bilayer QH systems electrons possess the four internal degrees of freedom, the spin and the layer (pseudospin). We denote the two layers as the “front” and “back” layers. The electron field operator in the bilayer QH systems is represented as $\Psi(\mathbf{x}) = (\psi_{\uparrow f}(\mathbf{x}), \psi_{\downarrow f}(\mathbf{x}), \psi_{\uparrow b}(\mathbf{x}), \psi_{\downarrow b}(\mathbf{x}))$. The physical operators in this system are expressed in terms of the following sixteen operators, the density operator and SU(4) isospin operators. In terms of the electron field operator $\Psi(\mathbf{x})$, they are given by

$$\begin{aligned} \rho(\mathbf{x}) &= \Psi^\dagger(\mathbf{x})\Psi(\mathbf{x}), \quad S_a(\mathbf{x}) = \frac{1}{2}\Psi^\dagger(\mathbf{x})\tau_a^{\text{spin}}\Psi(\mathbf{x}), \\ P_a(\mathbf{x}) &= \frac{1}{2}\Psi^\dagger(\mathbf{x})\tau_a^{\text{ppin}}\Psi(\mathbf{x}), \quad R_{ab}(\mathbf{x}) = \frac{1}{2}\Psi^\dagger(\mathbf{x})\tau_a^{\text{spin}}\tau_b^{\text{ppin}}\Psi(\mathbf{x}), \end{aligned} \quad (\text{A1})$$

where $a, b = x, y, z$ and

$$\tau_a^{\text{spin}} = \begin{pmatrix} \tau_a & 0 \\ 0 & \tau_a \end{pmatrix}, \quad \tau_x^{\text{ppin}} = \begin{pmatrix} 0 & \mathbf{1}_2 \\ \mathbf{1}_2 & 0 \end{pmatrix}, \quad \tau_y^{\text{ppin}} = \begin{pmatrix} 0 & -i\mathbf{1}_2 \\ i\mathbf{1}_2 & 0 \end{pmatrix}, \quad \tau_z^{\text{ppin}} = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & -\mathbf{1}_2 \end{pmatrix}, \quad (\text{A2})$$

with τ_a denoting the Pauli matrices. The operator $\rho(\mathbf{x})$ is the density operator, while the SU(4) isospin density operator $S_a(\mathbf{x})$, $P_a(\mathbf{x})$ and $R_{ab}(\mathbf{x})$ represent the spin density operator, the pseudospin density operator, and the R -spin density operator, respectively. On the other hand, the total Hamiltonian in this system is given by [2], $H = H_K + H_C + H_Z + H_{PZ}$, where H_K is the kinetic term which generates the Landau level, H_C the Coulomb interaction term, H_Z the Zeeman interaction term, and H_{PZ} the pseudo-Zeeman term composed of the tunnelling interaction term and the bias term, which represents the creation of the density-imbalanced configuration between the two layers. The Coulomb interaction is decomposed into the form $H_C = H_C^+ + H_C^-$, where $H_C^{+(-)}$ represents the SU(4) invariant (non-invariant) term.

We analyze the classical Hamiltonian density of $H = \int d^2x \mathcal{H}(\mathbf{x})$ by performing the lowest Landau level projection. We denote the generic lowest Landau level state as $|\mathfrak{S}\rangle$. We derive the form of $\mathcal{H}^{\text{cl}}(\mathbf{x}) = \langle \mathfrak{S} | \mathcal{H}(\mathbf{x}) | \mathfrak{S} \rangle$, which are represented by the classical density operators $\rho^{\text{cl}}(\mathbf{x}) = \langle \mathfrak{S} | \rho(\mathbf{x}) | \mathfrak{S} \rangle$, $S_a^{\text{cl}}(\mathbf{x}) = \langle \mathfrak{S} | S_a(\mathbf{x}) | \mathfrak{S} \rangle$, $P_a^{\text{cl}}(\mathbf{x}) = \langle \mathfrak{S} | P_a(\mathbf{x}) | \mathfrak{S} \rangle$, and $R_{ab}^{\text{cl}}(\mathbf{x}) = \langle \mathfrak{S} | R_{ab}(\mathbf{x}) | \mathfrak{S} \rangle$. Actually we express \mathcal{H}^{cl} in terms of the normalized SU(4) operators defined by $S_a^{\text{cl}}(\mathbf{x}) = \rho_{\Phi} S_a^{\text{cl}}(\mathbf{x})$, $P_a^{\text{cl}}(\mathbf{x}) = \rho_{\Phi} P_a^{\text{cl}}(\mathbf{x})$, and $R_{ab}^{\text{cl}}(\mathbf{x}) = \rho_{\Phi} R_{ab}^{\text{cl}}(\mathbf{x})$, by setting $\rho^{\text{cl}}(\mathbf{x}) = \rho_0$ due to the incompressibility of the QH state.

In the $\nu = 2$ bilayer QH system the SU(4) order parameters, which are the expectation values of the normalized SU(4) operators in the ground state, are given by [8]

$$\begin{aligned} \mathcal{S}_z^0 &= -\frac{\Delta_Z}{\Delta_0} (1 - \alpha^2) \sqrt{1 - \beta^2}, \\ \mathcal{P}_x^0 &= \frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha^2 \sqrt{1 - \beta^2}, \quad \mathcal{P}_z^0 = \frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha^2 \beta = \sigma_0, \\ \mathcal{R}_{xx}^0 + i\mathcal{R}_{yx}^0 &= -\frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha \sqrt{1 - \alpha^2} \beta e^{-i\omega}, \\ \mathcal{R}_{yy}^0 + i\mathcal{R}_{xy}^0 &= \frac{\Delta_Z}{\Delta_0} \alpha \sqrt{1 - \alpha^2} \sqrt{1 - \beta^2} e^{i\omega}, \\ \mathcal{R}_{xz}^0 + i\mathcal{R}_{yz}^0 &= \frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha \sqrt{1 - \alpha^2} \sqrt{1 - \beta^2} e^{-i\omega}, \end{aligned} \quad (\text{A3})$$

with

$$\Delta_0 \equiv \sqrt{\Delta_{\text{SAS}}^2 \alpha^2 + \Delta_Z^2 (1 - \alpha^2) (1 - \beta^2)}, \quad (\text{A4})$$

where Δ_Z is the Zeeman gap and Δ_{SAS} the tunneling gap; α, β ($|\alpha|, |\beta| \leq 1$) and ω are real parameters. The quantity σ_0 is the imbalanced parameter, representing the density difference between the front and back layers, and defined by $\sigma_0 = (\rho_0^{\text{f}} - \rho_0^{\text{b}}) / (\rho_0^{\text{f}} + \rho_0^{\text{b}})$, with $\rho_0^{\text{f(b)}}$ the electron density in the front (back) layer. Here the parameters α and β are determined by minimizing the classical Hamiltonian given by (B1), where the normalized isospin densities are in spatially homogeneous configurations. They satisfy the condition $(\mathcal{S}_a^0)^2 + (\mathcal{P}_a^0)^2 + (\mathcal{R}_{ba}^0)^2 = 1$. We demonstrate later that the parameter ω is associated with the NG mode in the CAF phase: See (B3).

It was shown [8] that for $\alpha = 0$, the ground state is the ferromagnetic phase, where only the spin is polarized as $\mathcal{S}_z^0 = 1$. On the other hand, for $\alpha = 1$ the ground state is the spin-singlet phase, where only the pseudospin is polarized as $\mathcal{P}_z^0 = \sigma_0$ and $\mathcal{P}_x^0 = \sqrt{1 - \sigma_0^2}$. The CAF phase is realized for $0 < \alpha < 1$.

From $2\mathcal{S}_a^{\text{f}} = \mathcal{S}_a^0 + \mathcal{R}_{az}^0$ and $2\mathcal{S}_a^{\text{b}} = \mathcal{S}_a^0 - \mathcal{R}_{az}^0$, the relations between the spin densities in the front and back layers are

$$\begin{aligned} \mathcal{S}_x^{\text{f}} &= -\mathcal{S}_x^{\text{b}} = \frac{1}{2} \frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha \sqrt{1 - \alpha^2} \sqrt{1 - \beta^2} \cos \omega, \\ \mathcal{S}_y^{\text{f}} &= -\mathcal{S}_y^{\text{b}} = -\frac{1}{2} \frac{\Delta_{\text{SAS}}}{\Delta_0} \alpha \sqrt{1 - \alpha^2} \sqrt{1 - \beta^2} \sin \omega, \\ \mathcal{S}_z^{\text{f}} &= \mathcal{S}_z^{\text{b}} = \frac{1}{2} \mathcal{S}_z^0. \end{aligned} \quad (\text{A5})$$

From the above equation, we see that in the CAF phase the antiferromagnetic correlation is built up between the two layers. Here the angle ω describes the orientation angle of the in-plane spin component. The order parameters (A3) are obtained from the ones with $\omega = 0$ by the spin rotation $\exp[iT_{z0}\omega]$. Furthermore, the Hamiltonian density (B1) is invariant under this rotation. Thus, the CAF phase is the $U_{T_{z0}}(1)$ spin rotational symmetry broken state.

At $\nu = 2$, four complex NG modes emerge due to the symmetry breaking pattern $\text{SU}(4) \rightarrow \text{U}(1) \times \text{SU}(2) \times \text{SU}(2)$. All the NG modes get gapped due to the presence of the Coulomb interactions, the Zeeman and pseudo-Zeeman terms. We analyze the system in the limit $\Delta_{\text{SAS}} \rightarrow 0$, where only one NG mode responsible to the interlayer phase coherence becomes gapless. In this limit the order parameters (A3) are reduced to

$$\begin{aligned} \mathcal{S}_z^0 &= |\sigma_0| - 1, \quad \mathcal{P}_z^0 = \sigma_0, \\ \mathcal{R}_{yy}^0 &= -\text{sgn}(\sigma_0) \mathcal{R}_{xx}^0 = \sqrt{|\sigma_0|(1 - |\sigma_0|)} \cos \omega, \quad \mathcal{R}_{xy}^0 = \text{sgn}(\sigma_0) \mathcal{R}_{yx}^0 = \sqrt{|\sigma_0|(1 - |\sigma_0|)} \sin \omega, \end{aligned} \quad (\text{A6})$$

with the relations

$$\frac{\Delta_Z}{\Delta_0} \sqrt{1 - \beta^2} = 1, \quad \frac{\Delta_{\text{SAS}}}{\Delta_0} = 1, \quad \alpha^2 = |\sigma_0|, \quad \sqrt{1 - \beta^2} = 0, \quad \beta = \text{sgn} \sigma_0, \quad (\text{A7})$$

which are obtained in the limit $\Delta_{\text{SAS}} \rightarrow 0$. We consider the case $\sigma_0 > 0$ explicitly, while the case $\sigma_0 < 0$ is similarly discussed. The order parameters (A6) imply that the SU(4) isospins are given by

$$\begin{aligned} \mathcal{S}_z(\mathbf{x}) &= \sigma(\mathbf{x}) - 1, \quad \mathcal{P}_z(\mathbf{x}) = \sigma(\mathbf{x}), \\ \mathcal{R}_{yy}(\mathbf{x}) &= -\mathcal{R}_{xx}(\mathbf{x}) = \sqrt{\sigma(\mathbf{x})(1 - \sigma(\mathbf{x}))} \cos \vartheta(\mathbf{x}), \quad \mathcal{R}_{xy}(\mathbf{x}) = \mathcal{R}_{yx}(\mathbf{x}) = -\sqrt{\sigma(\mathbf{x})(1 - \sigma(\mathbf{x}))} \sin \vartheta(\mathbf{x}), \end{aligned} \quad (\text{A8})$$

with all the others being zero, where $\sigma(\mathbf{x})$ and $\vartheta(\mathbf{x})$ are the canonical set of the NG mode. The ground-state expectation values of these fields must be $\langle \sigma(\mathbf{x}) \rangle = \sigma_0$ and $\langle \vartheta(\mathbf{x}) \rangle = \vartheta_0 = -\omega$. Since $\rho_\Phi \sigma(\mathbf{x})$ represents the density while $\vartheta(\mathbf{y})$ the angle variable, the following canonical commutation relation holds,

$$\rho_\Phi [\sigma(\mathbf{x}), \vartheta(\mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y}). \quad (\text{A9})$$

We refer the detailed derivation of the SU(4) isospins (A8) and the canonical commutation relation (A9) to Ref.[11].

Appendix B: Effective Hamiltonian for the NG Mode in the CAF phase

We next derive the effective Hamiltonian density and the dispersion relation. Apart from irrelevant constants, the basic Hamiltonian density for the ground state and the associated NG modes is given by [2]

$$\begin{aligned} \mathcal{H}^{\text{eff}} &= J_s^d \left(\sum (\partial_k \mathcal{S}_a)^2 + (\partial_k \mathcal{P}_a)^2 + (\partial_k \mathcal{R}_{ab})^2 \right) + 2J_s^- \left(\sum (\partial_k \mathcal{S}_a)^2 + (\partial_k \mathcal{P}_z)^2 + (\partial_k \mathcal{R}_{az})^2 \right) \\ &+ \rho_\phi \left[\epsilon_{\text{cap}} (\mathcal{P}_z)^2 - 2\epsilon_X^- \left(\sum_a (\mathcal{S}_a)^2 + (\mathcal{R}_{az})^2 \right) - (-\Delta_Z \mathcal{S}_z + \Delta_{\text{SAS}} \mathcal{P}_x \Delta_{\text{bias}} \mathcal{P}_z) \right], \end{aligned} \quad (\text{B1})$$

where $k = x, y$, Δ_{bias} the bias parameter, and

$$\begin{aligned} J_s &= J_s^+ + J_s^- = \frac{1}{16\sqrt{2}\pi} E_C^0, \quad J_s^d = J_s^+ - J_s^-, \quad J_s^d = J_s \left[-\sqrt{\frac{2}{\pi}} \frac{d}{\ell_B} + \left(1 + \frac{d^2}{\ell_B^2} \right) e^{d^2/2\ell_B^2} \text{erfc} \left(d/\sqrt{2}\ell_B \right) \right], \\ \epsilon_X &= \frac{1}{2} \sqrt{\frac{\pi}{2}} E_C^0, \quad \epsilon_X^\pm = \frac{1}{2} \left[1 \pm e^{d^2/2\ell_B^2} \text{erfc} \left(d/\sqrt{2}\ell_B \right) \right] \epsilon_X, \quad \epsilon_D^- = \frac{d}{4\ell_B} E_C^0, \quad \epsilon_{\text{cap}} = 4\epsilon_D^- - 2\epsilon_X^-, \end{aligned} \quad (\text{B2})$$

with $E_C^0 = e^2/4\pi l_B^2$ and d the layer separation.

We introduce the fluctuation fields $\delta\sigma(\mathbf{x})$ and $\delta\vartheta(\mathbf{x})$ around the ground state by

$$\sigma(\mathbf{x}) = \sigma_0 + \delta\sigma(\mathbf{x}), \quad \vartheta(\mathbf{x}) = \vartheta_0 + \delta\vartheta(\mathbf{x}). \quad (\text{B3})$$

Note that ϑ_0 is nothing but the the orientation angle of the in-plane spin component ω ($\times -1$) introduced in the main text: See (4). Substituting Eq. (A8) together with (B3) into the Hamiltonian density (B1), we obtain

$$\mathcal{H}^{\text{eff}} = \frac{J_\vartheta}{2} (\nabla \delta\vartheta)^2 + \frac{2J_\sigma}{\rho_0^2} (\nabla \check{\sigma})^2 + \frac{2\epsilon_{\text{cap}}^{\nu=1}}{\rho_0} \check{\sigma}^2, \quad (\text{B4})$$

where $\epsilon_{\text{cap}}^{\nu=1} = \epsilon_{\text{cap}} - 2\epsilon_X^-$, $\check{\sigma}(\mathbf{x}) = \rho_\Phi \delta\sigma(\mathbf{x})$ and

$$J_\sigma = 4J_s + \frac{(2\sigma_0 - 1)^2}{\sigma_0(1 - \sigma_0)} J_s^d, \quad J_\vartheta = 4J_s^d \sigma_0(1 - \sigma_0). \quad (\text{B5})$$

The Hamiltonian density (B4) is written in the second-quantized form when the canonical commutation relation (A9) is imposed. We introduce the annihilation and creation operators,

$$r_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left(\sqrt{G_{\mathbf{k}}} \check{\sigma}_{\mathbf{k}} + i \frac{1}{\sqrt{G_{\mathbf{k}}}} \delta\vartheta_{\mathbf{k}} \right), \quad r_{\mathbf{k}}^\dagger = \frac{1}{\sqrt{2}} \left(\sqrt{G_{\mathbf{k}}} \check{\sigma}_{\mathbf{k}}^\dagger - i \frac{1}{\sqrt{G_{\mathbf{k}}}} \delta\vartheta_{\mathbf{k}}^\dagger \right), \quad (\text{B6})$$

where $\check{\sigma}_{\mathbf{k}}$ and $\delta\vartheta_{\mathbf{k}}$ denoting the Fourier transforms of the fields $\check{\sigma}(\mathbf{x})$ and $\delta\vartheta(\mathbf{x})$, respectively, and

$$G_{\mathbf{k}} = \left(\frac{\lambda_\sigma}{\lambda_\vartheta} \right)^{1/2}, \quad \lambda_\sigma = \frac{2J_\sigma}{\rho_0^2} \mathbf{k}^2 + \frac{2\epsilon_{\text{cap}}^{\nu=1}}{\rho_0}, \quad \lambda_\vartheta = \frac{J_\vartheta}{2} \mathbf{k}^2. \quad (\text{B7})$$

From (A9), we obtain the commutation relations

$$[r_{\mathbf{k}}, r_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}'), \quad [\check{\sigma}_{\mathbf{k}}, \delta\vartheta_{\mathbf{k}'}^\dagger] = i\delta(\mathbf{k} - \mathbf{k}'). \quad (\text{B8})$$

By using (B6), the Hamiltonian density (B4) in the momentum space is diagonalized as

$$H_{\text{R}} = \int d^2k E_{\mathbf{k}} r_{\mathbf{k}}^\dagger r_{\mathbf{k}}, \quad (\text{B9})$$

where $E_{\mathbf{k}}$ is given by

$$E_{\mathbf{k}} = |\mathbf{k}| \sqrt{\frac{2J_{\vartheta}}{\rho_0} \left(\frac{2J_{\sigma}}{\rho_0} \mathbf{k}^2 + 2\epsilon_{\text{cap}}^{\nu=1} \right)} \simeq 2|\mathbf{k}| \sqrt{\frac{J_{\vartheta} \epsilon_{\text{cap}}^{\nu=1}}{\rho_0}} = \gamma|\mathbf{k}|. \quad (\text{B10})$$

Hence, the NG mode has the linear dispersion relation.

The wave length of the NG mode at $\omega_s \sim 10$ rad·MHz/T is estimated as

$$\lambda_s = \frac{2\pi}{k_s} = 2.90454 \times 10^7 \text{ \AA}, \quad (\text{B11})$$

where $k_s = \hbar\omega_s\gamma^{-1}$, and we have set $l_B = d = 230.967 \text{ \AA}$, or equivalently, $\rho_0 = 0.59669 \times 10^{-5} \text{ \AA}^{-2}$, while $\sigma_0 = 0.3676$, as typical values for QH samples, and $k_B = 1.3807 \times 10^{-23} \text{ J/K}$. The wavelength (B11) is about the same size as the sample size, $L \sim 100 \mu\text{m}$. Thus the long-wavelength approximation is valid in this case.

Since $\check{\sigma}(\mathbf{x})$ and $\delta\vartheta(\mathbf{x})$ are real fields and $G_{\mathbf{k}}$ are even function of \mathbf{k} , we obtain the relations $G_{\mathbf{k}} = G_{-\mathbf{k}}$, $\check{\sigma}_{\mathbf{k}}^\dagger = \check{\sigma}_{-\mathbf{k}}$ and $\delta\vartheta_{\mathbf{k}}^\dagger = \delta\vartheta_{-\mathbf{k}}$. Thus, by using them the phase field $\delta\vartheta(\mathbf{x})$ is described in terms of (B6) as

$$\begin{aligned} \delta\vartheta(\mathbf{x}) &= \int \frac{d^2k}{2\pi} e^{i\mathbf{k}\mathbf{x}} \left(-i\sqrt{\frac{G_{\mathbf{k}}}{2}} (r_{\mathbf{k}} - r_{-\mathbf{k}}^\dagger) \right) = \int \frac{d^2k}{2\pi} \left(\kappa_{\mathbf{x},\mathbf{k}} r_{\mathbf{k}} + \kappa_{\mathbf{x},\mathbf{k}}^* r_{\mathbf{k}}^\dagger \right), \\ \kappa_{\mathbf{x},\mathbf{k}} &= \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{i(\mathbf{k}\mathbf{x} - \frac{\pi}{2})}, \quad \kappa_{\mathbf{x},\mathbf{k}}^* = \sqrt{\frac{G_{\mathbf{k}}}{2}} e^{-i(\mathbf{k}\mathbf{x} - \frac{\pi}{2})}. \end{aligned} \quad (\text{B12})$$

In the long wave-length limit, we have $e^{ikx} \rightarrow 1$, and there is no \mathbf{x} dependence in $\kappa_{\mathbf{x},\mathbf{k}}$. In this limit we just write it as $\kappa_{\mathbf{k}}$.

Appendix C: Dicke Model in the CAF phase

We proceed to derive the interaction Hamiltonian between the nuclear spins and the NG mode in the CAF phase based on (3). We assume that only the nuclear spins in one of the layers are dynamically polarized. This is indeed the case in the experiment [12]. Thus we consider the interaction between nuclear spins in the front layer and the NG mode $\delta\vartheta$. From now on, we omit the pseudospin index in the spin density. First from (A8), we see that S_z is dynamically frozen, because the imbalanced field σ has a gap much larger than the thermal energy, and therefore, its excitation is suppressed. Thus the interaction between the nuclear spins and the NG mode is solely expressed by the spin-spin interaction for the in-plane component. For simplicity, we start from Eq. (A5) where any limits are not taken. By setting $-\omega \rightarrow \vartheta(\mathbf{X}_i) = \vartheta_0 + \delta\vartheta(\mathbf{X}_i)$, and expanding $\mathcal{S}_a(\mathbf{X}_i)$ in terms of $\delta\vartheta$ up to the linear order, we have

$$\mathcal{S}_x(\mathbf{X}_i) = S(\cos \vartheta_0 - \sin \vartheta_0 \cdot \delta\vartheta(\mathbf{X}_i)) + \mathcal{O}(\delta\vartheta^2), \quad \mathcal{S}_y(\mathbf{X}_i) = S(\sin \vartheta_0 + \cos \vartheta_0 \cdot \delta\vartheta(\mathbf{X}_i)) + \mathcal{O}(\delta\vartheta^2), \quad (\text{C1})$$

where $S = \Delta_{\text{SAS}} \alpha \sqrt{1 - \alpha^2} \sqrt{1 - \beta^2} / 2\Delta_0$ with $0 \leq S < 1$. Then, from (3) the hyperfine interaction for the in-plane component becomes

$$H_{\text{HF}} = \sum_{i=1}^N g(\cos \vartheta_0 I_i^x + \sin \vartheta_0 I_i^y) + \sum_{i=1}^N g\delta\vartheta(\mathbf{X}_i)(-\sin \vartheta_0 I_i^x + \cos \vartheta_0 I_i^y), \quad (\text{C2})$$

where $g = \tilde{g}S$. The first term in (C2) represents the in-plane Knight shift term and is much smaller compared with the Larmor frequency ω_s (see the discussion in Sec. II). With the same reason, we can neglect the interaction term between the z component of nuclear spins and that of electron spins. Hence we only retain the second term, representing the interaction between the nuclear spins and the NG mode. By introducing $I^\pm = I^x \pm iI^y$ and using (B12), we have

$$H_{\text{HF}} = \frac{g}{2} \sum_{i=1}^N (\tilde{I}_i^+ + \tilde{I}_i^-) \int \frac{d^2k}{2\pi} \left(\kappa_{\mathbf{X}_i,\mathbf{k}} r_{\mathbf{k}} + \kappa_{\mathbf{X}_i,\mathbf{k}}^* r_{\mathbf{k}}^\dagger \right), \quad (\text{C3})$$

where $\tilde{I}^\pm = e^{\mp i(\vartheta_0 + \pi/2)} I^\pm$ is the rotated in-plane nuclear spin. We just write them as I^\pm in the rest of this Appendix. By using the rotating-wave approximation, we obtain

$$H_{\text{SR}} = \frac{g}{2} \sum_{i=1}^N (I_i^+ R_i + I_i^- R_i^\dagger), \quad R_i = \int \frac{d^2 k}{2\pi} \kappa_{\mathbf{X}_i, \mathbf{k}} r_{\mathbf{k}}, \quad R_i^\dagger = \int \frac{d^2 k}{2\pi} \kappa_{\mathbf{X}_i, \mathbf{k}}^* r_{\mathbf{k}}^\dagger. \quad (\text{C4})$$

In the long-wavelength limit the i dependence disappears from R_i and R_i^\dagger . Thus we obtain the interaction Hamiltonian for the nuclear spins and the NG mode in the CAF phase,

$$H_{\text{SR}} = \frac{g}{2} \sum_{i=1}^N (I_i^+ R + I_i^- R^\dagger) = \frac{g}{2} (J^+ R + J^- R^\dagger). \quad (\text{C5})$$

By combining the Larmor-precision Hamiltonian (16), the effective Hamiltonian for the NG mode (B9), and the interaction Hamiltonian (C5), we have the Dicke model in the CAF phase.

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